New Method for Estimating Low-Earth-Orbit Collision Probabilities

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An unconventional but general method is described for estimating the probability of collision between an Earth-orbiting spacecraft and orbital debris. This method uses a Monte Carlo simulation of the orbital motion of the target spacecraft and each discrete debris object to generate an empirical set of distances, each distance representing the separation between the spacecraft and the nearest debris object at random times. Using concepts from the asymptotic theory of extreme order statistics, an analytical density function is fitted to this set of minimum distances. From this function, it is possible to generate realistic collision estimates for the spacecraft.

Nomenclature

$B()$	= beta function
$E(t_c)$	= expected time per collision
$F(x)$	= Weibull distribution function
$G(t_c)$	= distribution function of t_c
$g(t_c)$	= probability density function of t_c
m	= mode of a probability density function
N	= estimated number of collisions per unit time
R	= radius of sphere associated with a collision
r_c	= characteristic collision distance
T_c	= estimated time between collisions
t_c	= time per collision (characteristic encounter time)
V	= relative velocity
W	= $Vt_c/2R$
x	= distance
α	= location parameter of Weibull distribution
β	= scale parameter of Weibull distribution
$\Gamma()$	= gamma function
τ	= shape parameter of Weibull distribution

Introduction

DURING the past decade, increasing attention has been focused on the problem of quantifying the hazard posed by the growing amount of orbital debris. As spacecraft become larger and missions longer in duration, the hazard of potential collisions becomes an ever more serious concern.[1] Plans for permanently orbiting space stations of large dimension in low-altitude orbits particularly exacerbate this concern, as does the increasing crowding in such regions as the geosynchronous ring. It is therefore important to be able to estimate realistically the probability of collision for an orbiting object during an arbitrary interval, and such estimates should be based as much as possible on the actual environment that characterizes the object's specific orbit.

Considerable effort has already been expended on solving this problem. Several different methods have been developed that can be used to quantify the hazard from space debris. Although these methods differ in their details, they are generally based on a common underlying approach, i.e., representing the debris population by a spatial debris density characterized by the Poisson distribution, and then computing a collision probability based on this debris model. In this paper we discuss a method that implements a substantially different approach.

Approaches to Problem

An excellent survey and review of the principal methods that have been used by previous investigators to quantify the debris hazard is given in a recent book by Johnson and McKnight.[2] These methods generally convert the aggregate of discrete data describing the orbital debris population into some sort of continuous debris density to enable the calculation of collision probabilities. Unfortunately, in such a transformation of the discrete and dynamic debris environment into a static continuum, a large amount of kinematic information about the debris population is inevitably lost at the outset. This results in an attendant loss of both realism and detail when the probability of collision is estimated for an object in a specific orbit. Furthermore, it is difficult to model accurately the debris population, whose spatial density is both inhomogeneous and time varying, by a static spatial Poisson distribution, even when the model is multizoned. Although multizoned models may, at some cost in complexity, model the inhomogeneous nature of the debris population, its time-varying aspect remains unmodeled.

In view of the shortcomings associated with static spatial debris distributions, it is worth considering possible alternatives that would model the debris population differently. It would be advantageous to have a method of determining the collision probability directly from an arbitrary discrete debris population having specified orbits, while also being able to specify the target's orbit arbitrarily. Here the object designated as the target would represent the Space Shuttle, Space Station, or other object of concern, and the debris population would represent a real or hypothetical set of objects, each in its own orbit.

In this paper we develop such a method by taking an unconventional approach to the problem of determining orbital collision probability. Our approach bears little resemblance to previous methods and appears to offer some significant advantages, especially for the lower altitudes of particular concern to the Space Shuttle and Space Station programs. Our method entails applying the asymptotic theory of extreme order statistics[3] to the distribution of distances from the target object to the nearest debris object at random points in time. All of the debris objects are assumed to be moving in specified Keplerian orbits, as initially defined either by the U.S. Space Command (USSPACECOM) data base of tracked orbital objects or by some hypothetical data base. Thus, our model characterizes the debris population by a large set of Keplerian orbits, thereby retaining the kinematic information associated

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with each orbit; in addition, it retains the inhomogeneity of the spatial distribution of the debris. Since our target also has an arbitrary specifiable Keplerian orbit, we believe that our overall model embodies a substantial degree of realism.

Discussion

The underlying idea of our approach is quite simple, and can be easily understood by considering the actual situation for a target orbiting the Earth along with a large population of orbiting debris objects. In such a situation, it is clear that at a randomly chosen time there is always some debris object that is the closest one to the target at that particular moment. It follows that, by recording the distance from the target to the closest debris at a large number of different times, we could estimate the distribution of these distances. Such a distribution would therefore represent the relative fraction of time that the debris object nearest to the target is a given distance away. This distribution would taper off at small values of distance, and would vanish at a distance of zero. Sufficiently small values of distance would represent situations in which the nearest debris object is so close to the target as to collide with it. Hence, the problem of estimating the probability is largely equivalent to estimating the initial characteristics of the distribution function of the distance between the target and the nearest debris. Although this estimation may appear to require extrapolation (with its well-known dangers) of known data, in this application the extension of the distribution beyond the data region is strongly constrained by both physical and statistical considerations, as will be explained later. Thus, the hazards ordinarily associated with extrapolation are avoided.

The basic concept of our approach is, therefore, to observe the frequency of debris approach distances to a target on a larger, observable scale, and then to use statistical methods to extrapolate to frequencies of the virtually unobservable collision proximity distances in which we are interested. To obtain our observable data, we created a Monte Carlo simulation of the orbital dynamics of all USSPACECOM catalogued debris objects and an arbitrarily chosen target object. Using this simulation, we computed the distance between the target and the closest debris object at a random epoch. By repeating this process a large number of times, we obtained a set of minimum distances, each of which represents the closest approach to the target object at some time.

Theoretically, this set of minimum distances should be characterized by a particular distribution function (df), at least asymptotically.[3] Guided by this theory, we have fitted the data to the generalized extreme value distribution with very good results. Once the parameters of the distribution have been determined from the fit to the minimum distance data, the distribution is completely defined. Quantitative estimates of the fraction of an extended time interval, during which the target is within a specified distance of a debris object, can be directly determined from this distribution. To estimate the number of separate encounters during this interval, it is necessary to define the maximum proximity distance that entails a collision and to derive $E(t_c)$; here, t_c is a function of several known parameters, and it enables estimating the number of collisions in a given time period for a specified target. Each of these steps is elaborated in greater detail in the sections that follow.

USSPACECOM Tracked Debris Ephemeris Data

To define the orbital debris environment, we obtained ephemeris data generated by the USSPACECOM that represent the orbiting space objects tracked by this agency. We obtained these data for over a dozen different epochs from 1986-1988, thus representing the more recent populations of orbiting space objects. Obviously, these data do not include the countless smaller orbiting objects that cannot currently be tracked but that also pose a collision hazard. Nevertheless, the current population of over 7000 objects constitutes an excellent test environment and includes virtually all larger objects. Based on this known population, investigators have extrapolated to define the total environment, and have also assumed growth rates in the population to represent future debris environments. For our initial purposes, we have used only the known population of currently tracked objects. However, we have determined how to scale our results for an n-fold increase or decrease in the debris population, assuming that the additional objects are in similar orbits to the known population.

Monte Carlo Simulation

Implementing our method on a computer may be more time intensive than some other methods that have been employed, but it offers a very flexible and realistic representation of the debris environment since each debris orbit is defined by the orbital parameters catalogued in the USSPACECOM data base. The orbit of the target object is chosen by the user and can approximate that of any object of concern, such as the Space Shuttle or Space Station. The Monte Carlo simulation then assigns a random mean anomaly to the target object and each debris object, in this way specifying the location of each object on its orbit. The resulting sets of Kepler elements are then transformed to Cartesian coordinates, and the distance between the target and each debris object is calculated. The minimum of these distances is then stored along with the corresponding relative velocity. This simulation process is repeated at least several hundred times, with the mean anomaly of each debris object and the target reselected after the generation of each minimum distance. These mean anomalies are always selected from a uniform distribution of random numbers, thereby simulating the relative motion in time of the objects along their orbits. The end result of this entire process is a set of distances, each of which represents the closest approach distance between the target and the debris population at a random point in time.

The transformation of the Kepler elements to Cartesian coordinates necessitates solving Kepler's equation very frequently, and this increases the computation time involved. But we believe it is essential for realism in the generated data, since many debris orbits have appreciable eccentricities.

It may seem that the optimum method for generating a set of minimum distances would entail starting with a set of state vectors at a given epoch for both the debris objects and the target and actually propagating each of the existing 7000 or so orbits through time. Direct determination of the nearest object to the target at regular time intervals during the propagation would generate the desired minimum distance data set. In practice, this concept is both impractical and unnecessary. Huge quantities of computer time would be needed to propagate the thousands of different orbits, and the resulting data would differ insignificantly from that generated by the much faster Monte Carlo process already described. For whenever deterministic orbit propagation is carried out for an extended interval, perturbing forces such as atmospheric drag (never known with high accuracy) inevitably cause a rapid buildup of mostly along-track error, especially at low altitudes. Thus, the position along the orbit becomes highly uncertain, and soon is no better than a random guess.

As an indirect test of our assumption that orbit propagation can be replaced with the Monte Carlo simulation, we have applied our method to precisely those debris objects that are common to different epochs. Specifically, we took Kepler data from the USSPACECOM data base for two different epochs and eliminated all objects not common to both epochs. We then generated minimum distance sets for both epochs, fit the generalized extreme value distribution to the empirical data sets, and calculated quantitative collision frequencies for both epochs. Only minor changes in the collision hazard (for a low-altitude target) were observed for the different epochs. From this, we conclude that the change in the orbital configuration

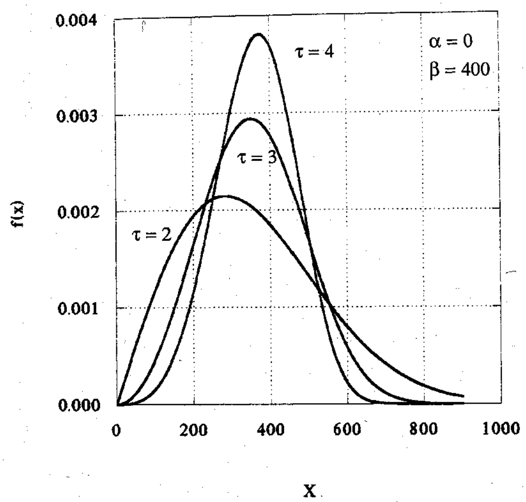

Fig. 1 Weibull probability density functions.

of the debris population as the orbits evolve in time is relatively unimportant, at least for the epoch separations tested and for low-altitude targets. This suggests that little, if anything, would be gained by propagating orbits to generate the minimum distance data, and that the Monte Carlo routine employed is a reasonable simulation of the relative motion of the debris objects and target.

Statistical Characterization of the Data

Once the minimum distance data has been generated by the Monte Carlo program, an analytical distribution is fitted to the empirical data. As already mentioned, the asymptotic theory of extreme order statistics indicates that the generalized extreme value distribution (GEVD) would be an appropriate choice for a limiting distribution of our data. A detailed exposition of this asymptotic theory is available in a recent monograph.[3]

There are only three possible specific forms of the GEVD: type I (Gumbel), type II, and type III (Weibull) distributions (this particular numbering scheme is not universally used). The fitting method of Hosking et al.,[4] which discriminates among the three, has consistently indicated that the Weibull yields the best fit to the data. This was expected on both statistical as well as physical grounds, largely because of the nature of the distribution of distance between any two orbiting objects.

The three parameters α, β, and τ completely define the Weibull distribution, and the Weibull df for minimum distances takes the following form

$$F(x) = 1 - \exp\{-[(x-\alpha)/\beta]^{\tau}\} \tag{1}$$

For small values of the argument, i.e., whenever $[(x-\alpha)/\beta] \ll 1$, $F(x)$ can be approximated as

$$F(x) \approx [(x-\alpha)/\beta]^{\tau} \tag{2}$$

Figure 1 shows the Weibull probability density function (pdf) for three different values of its shape parameter; the range from two to four for this parameter spans the range empirically obtained from our data. It is worth noting that the skewness of the Weibull distribution vanishes for $\tau \approx 3.6$; hence, symmetry is maximized for this value of the shape parameter and the distribution closely resembles the normal distribution.

For many applications, α vanishes, giving rise to the two-parameter Weibull distribution. A location parameter of zero forces the lower bound of the distribution to be zero; or, in other words, it restricts the distribution of the random variable to non-negative values only. It would naturally seem as though, in our application of the Weibull, we could simply use the two-parameter form, since it makes little sense to have a negative minimum distance. As an added test of our method, however, we have chosen to fit not only the two-parameter but also the three-parameter form of the Weibull. When applied to our data, the location parameter should be nearly zero. If, in fact, the value of the location parameter is not close to zero, it would raise questions about our method. Thus far in our investigation, we have indeed observed nearly vanishing values of the location parameters, thereby adding support to the applicability of our method to this type of data.

To convert our data to an analytical form, we have used several fitting techniques: the method of probability weighted moments,[4] the maximum likelihood method, and linear regression. Linear regression fits a straight line to the data when the latter is plotted, in effect, on Weibull probability paper (on which a Weibull distribution appears as a straight line). The first method provides a three-parameter fit, whereas the remaining methods provide two-parameter fits. Two linear regression fits were determined, one based on least squares (LS) and the other on least absolute deviations (LAD). The LAD criterion is much more robust with respect to outliers than is LS. In essence, these fitting routines simply estimate the parameters of the theoretical distribution, in this case a Weibull, which yield the best fit to the empirical data distribution. The four fitting methods all worked well. Among the four, the two in closest agreement were the maximum likelihood method and linear regression using LAD.

We have also used several goodness-of-fit statistics (such as the Anderson-Darling, Watson, and Cramer-von-Mises statistics[5-7]) to get an analytical measure of how well the Weibull distribution fits our empirical data. These goodness-of-fit statistics have generally indicated that the Weibull distribution does, in fact, yield a good fit to the data in almost all cases, especially when the number of data points is sufficiently large.

Characteristic Encounter Time

Once the analytical distribution of minimum distances is defined, it can be used to directly estimate the fraction of a given time interval during which the target is within a specified distance of the nearest debris object. This may be more clearly illustrated by utilizing a plot of the distributions: Fig. 2 is a plot of the analytical df and pdf, and also the empirical df, all for a target at an altitude of 450 km and an inclination of 28.5 deg. The analytical distributions are the smooth curves, and the empirical distribution is the jagged curve. As is evident from Fig. 2, the fit of the analytical df to the empirical df is very good. The left-hand scale corresponds only to the dfs; the pdf has been scaled to peak at unity only for illustrative clarity. For an estimate, therefore, one would choose a minimum distance on the horizontal axis, move up to the df, and read the corresponding value from the vertical scale. This number represents the fraction of any extended time interval during which the distance between the closest debris object and the target is less than or equal to the minimum distance initially chosen.

As described thus far, our method yields an estimate of the fraction of an extended time interval, during which the target is within a specified distance of another orbiting object. To estimate the number of separate encounters during this interval, it is necessary to determine a characteristic encounter time. As illustrated in Fig. 3, t_c is defined as the total time that a point A, moving with constant V past another point B, is within a distance R of B. Thus, t_c is the total time that point A is inside a sphere of radius R centered at B. Figure 3 illustrates that during the time t_c the shortest distance between points A and B is R_0. Given these assumptions and the previously derived result that R has a Weibull distribution with parameters β and τ, it can then be shown that the pdf of t_c is given by

$$g(t_c) = (\tau V W / 2R)\,[1 - W^2]^{(\tau/2-1)} \tag{3}$$

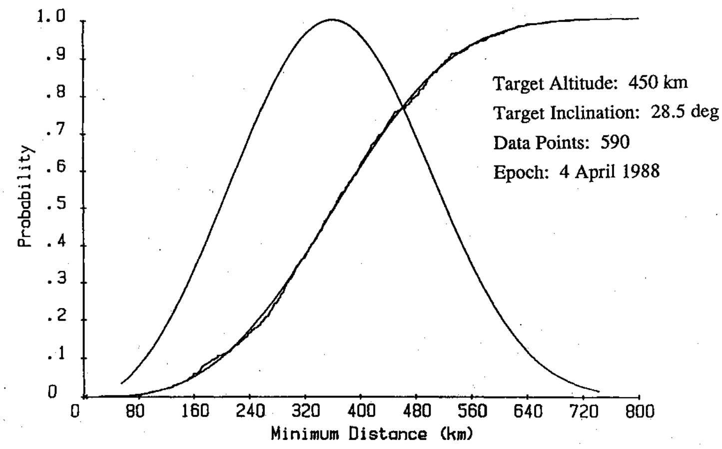

Fig. 2 Representative empirical and analytical Weibull distribution functions.

It follows that the expected value of t_c is given by

$$E(t_c) = (\tau R/V)B(\tau/2,3/2)$$
$$= (\tau R\pi^{1/2}/2V)\Gamma(\tau/2)/\Gamma[(\tau+3)/2] \qquad (4)$$

where $B()$ is the beta function and $\Gamma()$ is the gamma function. Furthermore, the df corresponding to Eq. (3) is given by

$$G(t_c) = 1 - (1 - W^2)^{\tau/2} \qquad (5)$$

The derivations of both Eqs. (3) and (4) are based on the assumption that the distance R is small compared to β; this assumption is realistic since encounter distances of interest are always much less than β. For extremely large encounter distances, a more general (and more complex) form of Eqs. (3) and (4) could be derived. Thus, the characteristic encounter time and its expected value are functions of the relative velocity during the encounter, the parameters of the extreme value distribution describing the minimum distance data, and the maximum distance characterizing an encounter that entails a collision ($R = r_c$). Since all of these parameters are known, it is possible to quantify the time per encounter and, thus, the number of encounters as well. Although the derivation of Eqs. (3) and (4) is based on point objects, the spherical radius R may, of course, be assigned a value appropriate to the sizes of the actual target and debris objects.

Collision Probability and Time Between Collisions

Given a characteristic collision distance and a characteristic collision time, it now follows that the estimated number of collisions per unit time is given simply by

$$N = F(r_c)/E(t_c) \qquad (6)$$

where $F()$ and $E()$ were given earlier in Eqs. (1), (2), and (4). Substituting Eqs. (2) and (4) into Eq. (6) yields

$$N = (1+1/\tau)(V/\beta\pi^{1/2})\{\Gamma[(\tau+1)/2]/\Gamma(\tau/2)\}(r_c/\beta)^{\tau-1} \qquad (7)$$

The estimated time between collisions is simply the reciprocal of N, and is, therefore, given by

$$T_c = (1+1/\tau)^{-1}(\beta\pi^{1/2}/V)\{\Gamma(\tau/2)/\Gamma[(\tau+1)/2]\}(r_c/\beta)^{1-\tau} \qquad (8)$$

With Eqs. (7) and (8), we have therefore derived explicit expressions for quantifying the collision hazard posed by orbiting debris. These equations also manifest the functional dependence of N and T_c on the relevant parameters β, τ, V, and r_c; obviously the most complex dependency is on τ. It is worth noting that if the debris were homogeneously distributed in space, then τ would have a value of 3. For $\tau = 3$, Eq. (8) becomes simply

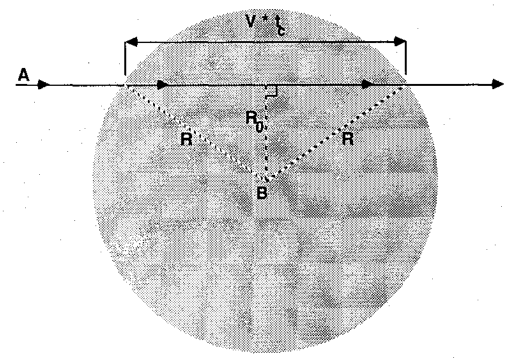

Fig. 3 Time per target-debris encounter.

$$T_c = (3\pi\beta/8V)(\beta/r_c)^2 \qquad (9)$$

As discussed in the ensuing section, the actual values determined for τ tend to cluster about 3: the interval from 2 to 4 includes all of the values thus far determined.

Results

Although our emphasis in this paper is to describe our method for quantifying the orbital debris collision hazard, we have also used our method to generate some representative numerical results. We have mainly concentrated on generating results of interest to the Space Station and Shuttle programs. In addition to these results, we have generated some results that are useful for comparison with those previously reported in the literature; the latter, however, are not easy to find in any substantial quantity.

The results summarized in Table 1 have been generated based on the USSPACECOM data base of tracked orbiting objects at the epoch of 88095 (April 4, 1988). The first two columns of Table 1 specify the altitude and inclination of the target's circular geocentric orbit. The third column gives the average relative velocity between the target and the nearest debris object; as mentioned earlier, for every minimum distance determined, the corresponding relative velocity is also computed. The fourth column specifies the number of minimum distance points generated by the Monte Carlo simulation of the relative motion. The next two columns indicate the values of the Weibull distribution parameters determined for the analytical fits of the minimum distance data sets. Finally, the last column gives the estimated time between collisions that the target would experience from the particular debris population assumed. The value of the characteristic collision distance used was 50 m, corresponding roughly to Space Station dimensions. The target orbit altitudes of 400–500 km and

Table 1 Representative times between collisions

Target altitude, km	Target inclination, deg	Relative velocity, km/s	Number of points	Shape parameter τ	Scale parameter β	Time between collisions, yr
300	28.5	8.13	1300	3.658	505.6	95800
400	0	8.52	1200	3.201	452.9	994
400	28.5	8.14	900	3.277	447.4	1980
400	60	8.16	812	3.367	409.2	3290
400	90	10.7	935	3.442	422.7	5490
400	120	11.9	1200	3.372	414.6	2477
400	150	12.3	1215	3.478	437.5	7390
400	180	12.2	895	3.330	451.9	2210
450	28.5	8.06	590	3.102	408.3	313
500	0	8.63	1145	2.986	407.0	103
500	28.5	8.23	1578	3.008	395.5	121
500	60	8.50	1205	3.061	359.5	139
500	90	10.8	720	2.980	367.2	57.3
500	120	11.9	736	3.116	374.0	183
500	150	12.3	1080	3.032	387.0	93.9
500	180	12.1	812	3.046	400.8	120
600	28.5	8.61	973	3.071	356.6	147
700	28.5	8.95	800	2.878	334.6	21.6
800	28.5	8.94	2013	2.795	321.1	9.38
975	28.5	8.93	1000	2.731	331.5	5.89

inclination of 28.5 deg also correspond to those intended for Space Station.

The estimated times between collisions given in Table 1 do not appear to differ radically from those given by other methods, although strictly comparable results are scarce. In agreement with others, our results also show a sharply increasing hazard as a function of increasing altitude, at least up to the well-known peak concentration of larger debris at 800–1000 km altitude. It is interesting to observe that the values of the Weibull shape parameter τ cluster on both sides of $\tau = 3$. Since, as noted earlier, a homogeneous distribution of debris would imply that $\tau = 3$, the actual deviations from this value can be interpreted as a measure of the nonhomogeneity of the debris environment for a particular target orbit. Values above 3 generally occur at very low altitude, whereas values below 3 occur at higher altitudes. At an altitude of 500 km, this parameter appears to nearly equal 3.

The values of the Weibull scale parameter tabulated in Table 1 generally range between 300–500 km. Since the mode of the Weibull distribution is given by

$$m = \beta(1 - 1/\tau)^{1/\tau} \tag{10}$$

it follows that the modes corresponding to the parameter values in Table 1 would tend to be slightly less than β. Since the mode is the most probable value of the distribution, we can conclude that at any given time the nearest tracked debris object is probably less than 300–500 km from the target.

The β parameter is also important for scaling results for a larger or smaller debris population. If a given debris population is assumed to change in size by a factor of n, while retaining essentially the same distribution of orbits, then it can be shown that the only change to Eqs. (7–9) is to divide β (wherever it occurs) by a factor of $n^{1/\tau}$. This result enables the estimation of the time between collisions for other than USSPACECOM tracked debris populations, provided the assumption of similarity distributed orbits is considered realistic. Thus, the effects of future growth in the debris population can be estimated, as can the effect of the much larger population of smaller untracked debris.

Conclusion and Future Directions

The method we have described for quantifying the orbiting debris collision hazard has a number of distinct advantages, most of which we have already pointed out in the foregoing sections. For a known or hypothetical debris population and a known target orbit, it enables a rather direct, intuitive, and flexible approach to estimating the probability of collisions. This approach can cope easily with an arbitrary target orbit and an inhomogeneous debris distribution. Our method lends itself well to scaling for a debris population that may either change in size in the future, or that may be underestimated because of current observability limitations. It can be implemented readily on a small computer, just as all of our computational work was done on personal computers of modest capability.

As mentioned previously, we have chosen to emphasize our methodology in this paper rather than specific numerical results. If specific results are needed for any particular application, it might be well to implement our method on a faster computer that would allow the generation of larger, and hence more statistically definitive, data sets.

In a possible future effort it would be useful to develop the means of deriving the distribution of the time between collisions, rather than just a point estimate, as given in Eq. (8). This would necessitate first determining the distribution of V, since T_c is functionally dependent on V. It would also involve using the distribution of the characteristic encounter time given by Eq. (3), rather than only its expected value.

Finally, we would note that although our approach to the problem of quantifying the orbital collision hazard differs significantly from previous methods, it is related to certain earlier investigations. The initial application of the asymptotic theory of extreme order statistics to the orbital collision problem appears to be that of McCormick[8] and Vedder, who applied the theory to the problem of predicting collision probability in the geosynchronous ring. Subsequently the theory was applied to assess the collision hazard of colocated geostationary satellites by Liebold[9] and by Härting et al.[10] These earlier investigations all used orbit propagators, rather than Monte Carlo simulations, to generate their minimum distance data. Furthermore, their results were limited to the geosynchronous region, and did not extend to the lower orbital altitudes of particular concern in our present investigation.

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